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Original Research Article

Non Linear Damped Oscillation of A Doubly Curved Shallow Shell on an Elastic Foundation

Subhash Chanda[1], Saptaparna Chanda[2]

[1]Associate Professor, A. C. College, Jalpaiguri, West Bengal, India
[2]Asstt. Prof., NIIS, Cooch Behar, West Bengal, India

Corresponding Author: Subhash Chanda

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ABSTRACT

The present paper aims at finding the damping effect on the non-linear vibrational modes of a doubly curved shallow shell placed on an elastic foundation by using the method of constant deflection contour lines. The results obtained are compared with the available results and the method yields very satisfactory results which are in excellent agreement with the reference values.

Keywords: Elastic sub-grade, Planforms, Pasternak foundation

INTRODUCTION

The nonlinear dynamic response of shallow shells has been investigated extensively by many authors. [1-8] Their works are mainly confined to the analysis of spherical shells having circular planforms without considering the interaction with the supporting elastic subgrade. The effect of the supporting medium in the use of soft filaments in aerospace structures, building activities in cold regions, foundation of heavy duty machines, under water and embedded structures needs special consideration to analyze. There is a recent study by Nath and Jain [5] in which an axisymmetric shallow spherical shell supported on an elastic foundation and undergoing moderately large deformations has been investigated by using Chebyshev series and Houbolt [8] technique. Very recently Nath, Mahrenholtz and Verma [1] studied the nonlinear dynamic response of a doubly curved shallow shell on an elastic foundation neglecting the effect of damping of the foundation. In what follows here an investigation is described of the effect of damping softness and mass of the elastic sub grade on the response of a doubly curved shallow spherical shell continually supported by Pasternak foundation [6] and undergoing moderately large dynamic deformations. The geometry and coordinate system are shown in figure (1).

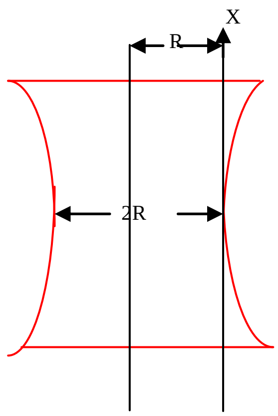


Figure 1-a: Geometry of the shell with negative Gaussian curvature.

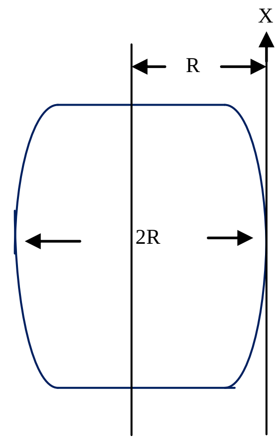


Figure 1-b: Geometry of the shell with positive Gaussian curvature.

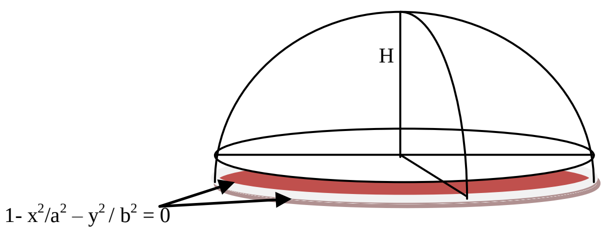


Figure-1-c: Shallow dome upon an elliptic base.

The strain components at the mid-plane for a thin shallo shell of principal curvatures R_x and R_y are

$$\begin{aligned} \varepsilon_x &= \{u_{,x} + (1/2)\, w^2_{,x}\} - R_y\, w_{,xx} \\ \varepsilon_x &= \{v_{,y} + (1/2)\, w^2_{,y}\} - R_x\, w_{,yy} \\ \varepsilon_{xy} &= \{u_{,y} + v_{,x} + w_{,x}\, w_{,y}\} \end{aligned} \qquad (1)$$

where, u, v, w are the displacement components in the X-,Y-,& Z- directions respectively. ε_x, ε_y, ε_{xy} are the strain components. The Pasternak model [6] for the elastic subgrade is assumed. Considering the normal inertia and damping of the foundation, one may express the interaction force on the shell at any time't' as

$$Q = kw - G'\Delta^2 w + k_d\, w_{,t} + \rho_f\, h_f\, w_{,tt} \qquad (2)$$

Where k, G', ρ_f, h_f and k_d are the normal stiffness, shear stiffness, density, thickness and damping constant respectively of the foundation. Neglecting the In-plane and rotator inertia terms, the governing equations of motion for the isotropic thin spherical shell undergoing moderately large deflection may be expressed in terms of the displacement components as

$$u_{,xx} + (½)(1-\nu)\, u_{,yy} + (½)(1+\nu)\, v_{,xy} - (R_y + \nu R_x)\, w_{,x} + [w_{,xx} + (½)(1-\nu)\, w_{,yy}]\, w_{,x} + (½)(1+\nu)\, w_{,xy}\, w_{,y} = 0 \qquad (3)$$

$$v_{,yy} + (½)(1-\nu)\, u_{,xx} + (½)(1+\nu)\, u_{,xy} - (R_x + \nu R_y)\, w_{,y} + [w_{,yy} + (½)(1-\nu)\, w_{,xx}]\, w_{,y} + (½)(1+\nu)\, w_{,xy}\, w_{,x} = 0 \qquad (4)$$

$D\ \Delta^4 w\ - (Eh/1-\nu^2)[\{u_{,X} + (1/2) w^2_{,X} - R_Y\ w\} + \nu\{v_{,y} + (1/2) w^2_{,y} - R_x w\}]\{R_Y + w_{,xx}\} - (Eh/1-\nu^2)[\{v_{,y} + (1/2) w^2_{,Y} - R_x\ w\} + \nu\{u_{,Y} + (1/2) w^2_{,x} - R_y w\}]\{R_x + w_{,yy}\} - (Eh/(1+\nu)\{u_{,y} + v_{,x} + w_{,x}\ w_{,y}\} w_{,xy} + Q + \rho_f\ h_f\ w_{,tt} = 0$ (5)

Where, $D = E\ h^3/12(1-\nu^2)$ and other parameters are defined earlier. Since ,normal displacements (w) is of prime interest, the in-plane displacements 'u' and 'v' are eliminated through integrations by proper choice of the displacement functions compatible with their boundary conditions.

Now, using the theory of constant deflection contour lines and taking into account that'w' and its derivatives with respect to 'u' are constants on the line u=constant,the equation (5) reduces to the following integro-differential equation:

$w_{,uuu} \oint R.ds + w_{,uu} \oint F.ds + w_{,u} \oint G.ds - (12/h^2)\iint [(1/2)\{w^2_{,x} + w^2_{,y}) (\nu R_X + R_Y)w](R_Y + w_{,XX}) dx\,dy - (12/h^2)\iint [(1/2)\{w^2_{,x} + w^2_{,y}) (\nu R_y + R_x)w](R_x + w_{,yy}) dx\,dy - 12(1-\nu)/h^2 \iint [(w_{,x}\ w_{,y}\ w_{,xy}] dx\,dy - \iint (kw/D)\,dx\,dy - (G'/D)\iint \Delta^2 w\,dx\,dy + k_d/D \iint w_{,t}\,dx\,dy + (\rho_f\ h_f + \rho\ h)/D \iint w_{,tt}\,dx\,dy = 0$ (6)

Illustration:

To solve equation (6) and to investigate the effect of damping one may consider the shell with elliptic planform and with semi major and semi minor axes 'a' & 'b', respectively. From symmetry consideration one may assume that the lines of equal deflection form a family of similar and similarly situated ellipses starting from the outer boundary as one these lines. Therefore, the equation of lines of equal deflection may conveniently be taken of the form:

$$u(x,y) = 1 - x^2/a^2 - y^2/b^2 \quad (7)$$

With this expression of u(x,y) the values of R ,F , G and t_1 are as follows:

$$R = -8D/p^3$$
$$F = 4Dp[(1-u)/a^2 b^2 - 3(x^2/a^6 - y^2/b^6)]$$
$$G = 2D(1-\nu^2)(1/a^2 - 1/b^2) p^5 (x^2/a^4 - y^2/b^4)(1-u) \quad (8)$$
$$t_1 = 4/p^2$$

where, $p^2 = [x^2/a^4 + y^2/b^4]^{-1}$

Substituting equations (7) & (8) in equation (6) one gets,

$(1-u)^2 w_{,uuu} - 2(1-u) w_{,uu} - H_1 \int_1^u (1-u) w^2_{,u}\,du + H_2 \int_1^u (1-u)^2 w^3_{,u} + H_3 \int_1^u (1-u)^2 w^3_{,u}\,du + H_4 \int_1^u w\,du + H_5 \int_1^u (1-u)\ w.w_{,u}\,du - H_6 \int_1^u w\,du - H_7 \int_1^u (1-u) w_{,u}\,du\ H_8 \int_1^u w_{,t}\,du - H_9 \int_1^u w_{,tt}\,du = 0$ (9)

Where, $H_1 = [3(a^2+b^2)(1+\nu)]/h^2(3a^4+2a^2b^2+3b^4)$;

$H_2 = [6(a^2+b^2)]/h^2(3a^4+2a^2b^2+3b^4)$;

$H_3 = [6(3\nu-1)a^2b^2]/h^2(3a^4+2a^2b^2+3b^4)$;

$H_4 = [(6a^4b^4)(R^2{}_1 + R^2{}_2 + 2\nu R_1 R_2)]/h^2(3a^4+2a^2b^2+3b^4)$;

$H_5 = 6a^2b^2(K_2 a^2 + K_1 b^2)]/h^2(3a^4+2a^2b^2+3b^4)$;

$H_6 = [k\,a^4b^4]/2D(3a^4+2a^2b^2+3b^4)$;

$H_7 = [G'a^2b^2(a^2+b^2)]/2D(3a^4+2a^2b^2+3b^4)$;

$H_8 = [k_d a^4 b^4]/(3a^4+2a^2b^2+3b^4)$;

$H_9 = [a^4b^4(\rho_f\ h_f + \rho\ h)/2D(3a^4+2a^2b^2+3b^4)$;

To solve equation (9) one may assume,

$w = \sum F(t)\, w_0\, u^j$, where the limits of summation are from 1 to ∞ (10)

Clearly, w satisfies the clamped edge boundary conditions,

$$W = (dw/du) = 0 \quad \text{at} \quad u=0$$

Now, considering the first two terms of the series and substituting equation (10) in equation (9) and applying Galerkin's technique one obtains:

$$(d^2/dt^2)\ F(t) + C\,[d\,F(t)/dt] + C_1\,F(t) + C_2\ w^2{}_0\,F^3(t) = 0 \qquad (11)$$

Where, $C = H_8\ /\ H_9$; $C_1 = -\,[\,0.6 + 0.6\,H_4 + 0.34\ H_5 + 0.6\,H_6\ _-\ 6.86H_7\ ]/\ H_9$

$C_2 = [\,21.45\ H_2 + 1.06\ H_3 - 2.79\ H_1\ ]/H$

Equation (11) is well known and its solution is obtained as

$$F(t) = a_0\ e^{-Ct}\ \ \text{Sin}\,[\,Ct\,\{\ 1 + (3/8)\,(C_2/C_1)\,a^2{}_0\,w^2{}_0\ \} + \Phi_0\ ] \qquad (12)$$

The time periods of the nonlinear and linear oscillations are

$$T^* = 2\pi/\ \{\ 1 + (3/8)\,(C_2/C_1)\,a^2{}_0\,w^2{}_0\ \}$$

$$T = 2\pi/\,C_1 \quad .$$

Thus, $$T^*/T = \{\ 1 + (3/8)\,(C_2/C_1)\,a^2{}_0\,w^2{}_0\ \}^{-1} \qquad (13)$$

Numerical Results:

Numerical results have been computed for first modal vibration of doubly curved shallow shell placed on elastic foundation with and without consideration of the effect of damping for different values of the foundation parameters ($k\,a^4/D$) and those are presented in tables and the effect of damping has been shown in figures (I – V). The computations are made with material constants, $\nu = 0.3$, $k\,a^4/D = 0, 100, 200, 400$ and $R_1 = R_2 = 5$.

TABLE- I: The ratio of nonlinear to linear time periods for damped vibrations of doubly curved shallow shells.

Time't'	Foundation parameter ($k\,a^4/D$)	T*/T →				
		W_0/h → 0.0	0.25	0.5	0.75	1.0
0 Sec.	0	1.0000	0.9825	0.9333	0.8615	0.7778
5 Sec.	0	1.0000	0.9868	0.9375	0.8677	0.7802
10 Sec	0	1.0000	0.9894	0.9390	0.8700	0.7865
0 Sec.	100	1.0000	0.9806	0.9267	0.8488	0.7596
5 Sec.	100	1.0000	0.9814	0.9283	0.8504	0.7625
10 Sec	100	1.0000	0.9823	0.9300	0.8522	0.7645
0 Sec.	200	1.0000	0.9783	0.9186	0.8367	0.7382
5 Sec.	200	1.0000	0.9790	0.9196	0.8379	0.7396
10 Sec	200	1.0000	0.9800	0.9208	0.8393	0.7412
0 Sec.	400	1.0000	0.9716	0.8964	0.7918	0.6815
5 Sec.	400	1.0000	0.9724	0.8971	0.7930	0.6830
10 Sec	400	1.0000	0.9730		0.7938	0.6843

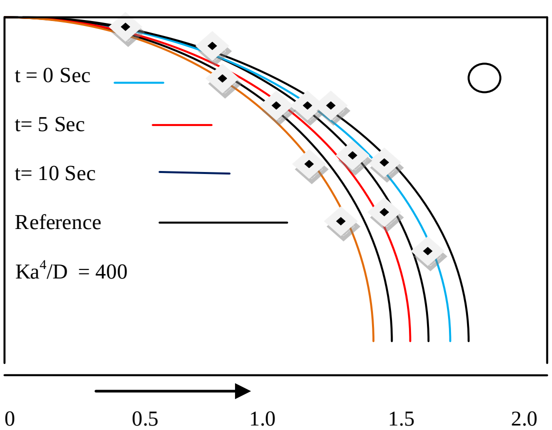


Figure-I: Effect of Damping on the nonlinear time periods of a doubly curved shallow shell.

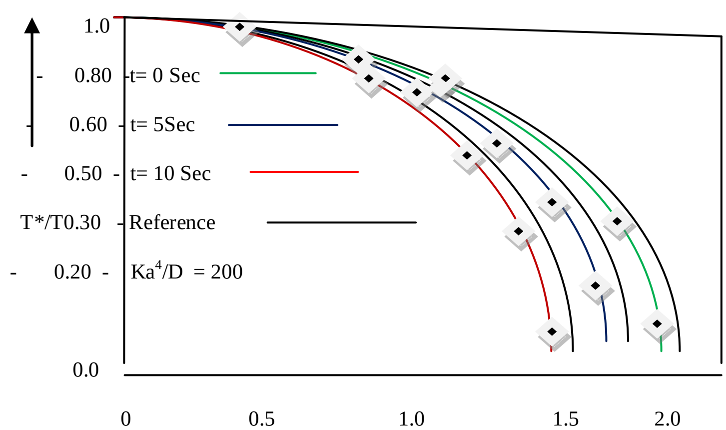
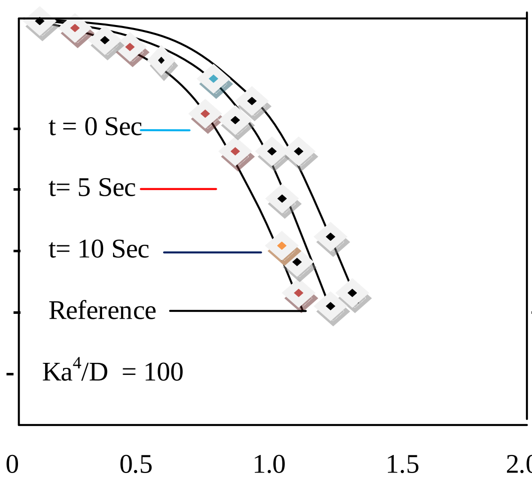


Figure-II: Effect of Damping on the nonlinear time periods of a doubly curved shallow shell.



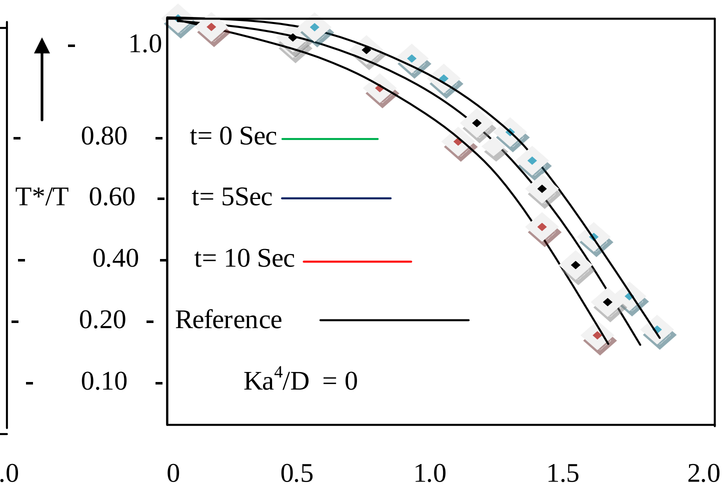


Figure-III: Effect of Damping on the nonlinear time periods of a doubly curved shallow shell.

Figure-IV: Effect of Damping on the nonlinear time periods of a doubly curved shallow shell.

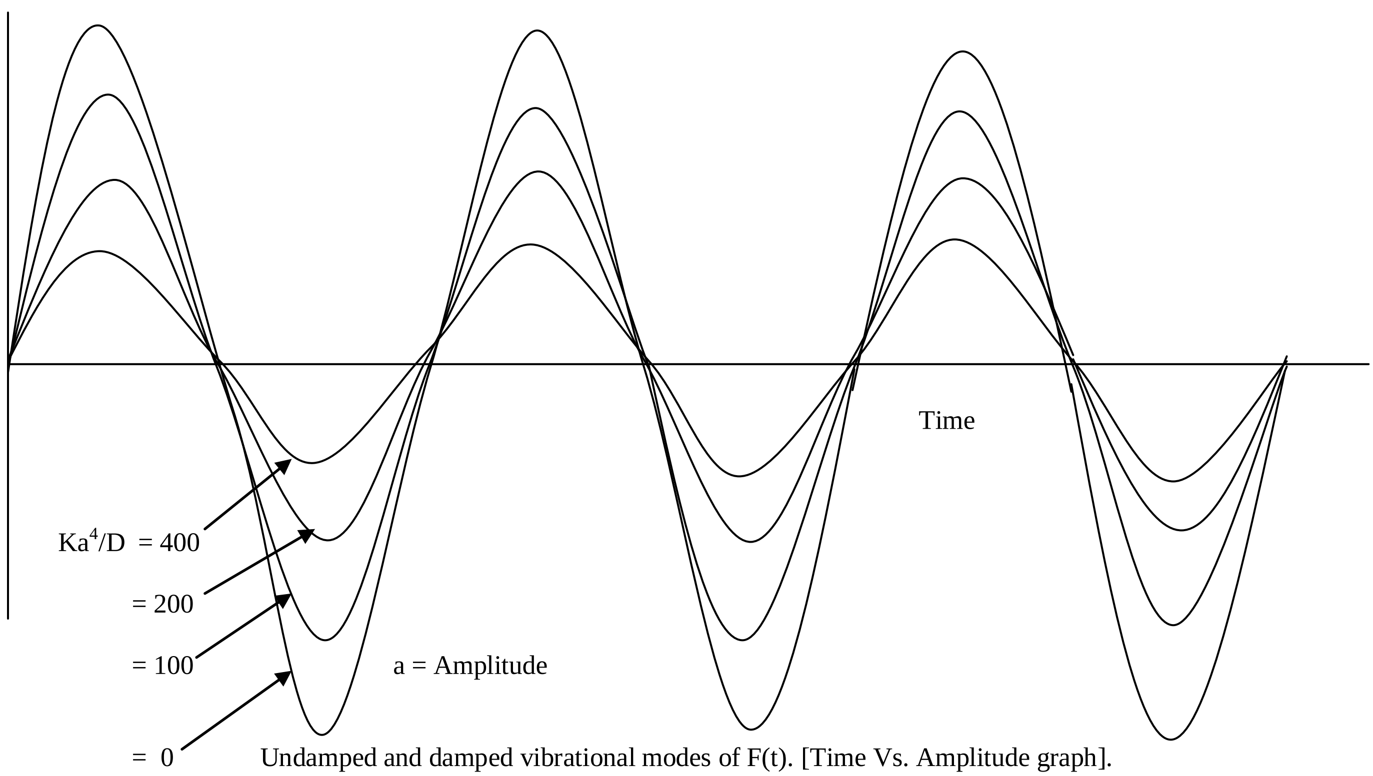


Figure-V: Non-linear damped oscillations of a doubly curved shallow shell on an elastic foundation.

OBSERVATIONS AND CONCLUSIONS

The numerical results for various foundation parameters and time are presented in table- I. Also, those are depicted in figures I-V. Figures I-IV showed the effect of damping on the nonlinear oscillations of a doubly curved shallow shell. It is found that in all cases the damping force enhances the magnitudes of the relative time periods which are in conformity with reality.

Figure - V shows the non-linear vibrational modes with and without damping for different values of the foundation parameters. It is evident from figure - V, that the amplitude of vibration is decaying continually with time and it is also observed that for a higher value of Ka^4/D, the initial amplitude of vibration decreases. The effect of damping gradually diminishes as the value of the foundation parameter increases, though the initial amplitude of vibration decreases with that parameter. It is to be noted here that the ratio of the nonlinear to linear time periods depends on the values of C_1, C_2 and C_3 in presence of damping force and depends only on the values of C_2 and C_3, in absence of damping force. So, it may be concluded that the effect of the foundation plays a great role in vibration during damping and increases with the increase of the mass of the sub-grade.

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