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# A New Method of Dividing Fractions

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## ABSTRACT

Fractions, an extension of the number system, are difficult to understand, especially when it comes to division. The challenges are particularly evident in the conventional 'invert and multiply' method, which lacks conceptual meaning. The common denominator method, though less commonly used, fosters a deeper conceptual understanding. This paper discusses the common denominator method and its advantages. Additionally, a new method, the cross-multiplication method, is proposed. This method originates from the lowest common multiple approach and is justified with an illustration. The advantages of the cross-multiplication method over the conventional method are highlighted. Finally, recommendations are made to incorporate the new method into teaching while abandoning the 'invert and multiply' method, as it encourages memorization without conceptual understanding.

***Keywords:*** Fractions, denominator method, cross-multiplication method

## INTRODUCTION

Division of fractions forms the foundation for a higher level of mathematical learning and reasoning, such as algebra (Lo & Luo, 2012). Therefore, it is essential to establish a conceptually sound approach to dividing fractions.

The "Invert and multiply" rule for dividing fractions is problematic because it lacks a conceptual basis. It is merely a procedural heuristic designed to yield correct results without justification. It is a heuristic that has been developed for purposes of giving a result but it has no justification. Many teachers perceive Mathematics as a set of procedures or tricks to solve mathematical problems quickly. Mnemonics such as "All Science Teachers are Crazy" for identifying quadrants in which each of the basic trigonometrical ratios is positive, "BODMAS", for the order of carrying mathematical operations, "My Dear Aunt Sally" to remember the order of operations and "Keep, Change, Flip" to divide fractions, are some of such tricks. Researchers argue that memorizing these tricks and applying them correctly does not equate to understanding the underlying concepts. In Principles to Actions (2014), the National Council of Teachers of Mathematics (NCTM) noted that mastering the procedures without the conceptual understanding of mathematics is one reason why mathematics education in the United States of America has not advanced in recent years.

Both pupils and teachers lack a conceptual understanding of the 'invert and multiply' algorithm. Lee et al. (2023) contend that while teachers possess procedural knowledge, they often lack a solid understanding of fraction division. Whenever teachers and pupils discuss fraction division, the 'invert and multiply' technique is the primary method that comes to mind. However, this method provides an answer without developing a conceptual foundation. This paper aims to popularize the lesser-known common denominator method for dividing fractions by demonstrating how it aids conceptual understanding rather than promoting rote memorization. Additionally, a simpler alternative method, the cross-multiplication method, is proposed and justified. The remainder of the paper is organized into five sections: the common denominator approach, the lowest common multiple approach, the cross-multiplication approach, the conclusion, and recommendations.

## LITERATURE REVIEW

### The common Denominator Approach

In general, division of fractions deals with two fractions with different denominators. The common denominator approach expresses these two fractions with a common denominator. A key advantage of this method is that it enables learners to conceptualize fraction division through visual representations. Research indicates that teachers struggle with using visual representations to explain fraction division (Borko et al., 1992; Jansen & Hohensee, 2016). The common denominator method facilitates such visual representations, making the concept more tangible. The example below shows how this method orks.

Example: Required to divide $\frac{3}{4}$ by $\frac{2}{3}$ we write $\frac{3}{4} \div \frac{2}{3}$. These two factions have different denominators. The common denominator of our fractions is 12. When we convert the two fractions to those with denominator 12 we have $\frac{9}{12} \div \frac{8}{12}$. When denominators are identical we then consider only numerators. It is also acceptable to divide numerators on their own and denominators on their own like what we do in the multiplication of fractions. This can only be done after equating denominators. The visual representation in figure 1 shows how we represent the two fractions of interest.

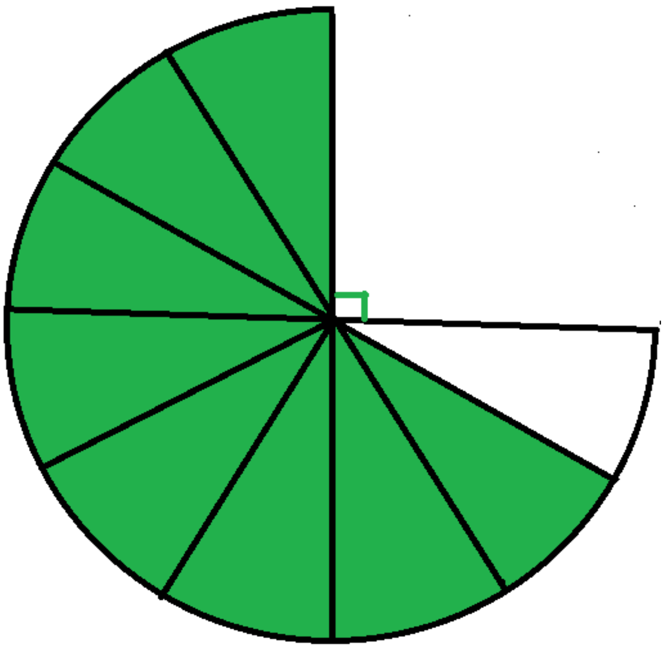

**Figure 1: A representation of $\frac{3}{4}$ which is $\frac{9}{12}$ and $\frac{2}{3}$ which is $\frac{8}{12}$**

Figure 1 shows $\frac{3}{4}$ of a circle. The complete circle is the whole. The $\frac{3}{4}$, which is the dividend, is shown as $\frac{9}{12}$, 8 shaded and 1 unshaded. The divisor, which is $\frac{2}{3}$, is represented by the shaded 8 parts, that is, $\frac{8}{12}$. By dividing $\frac{3}{4}$ by $\frac{2}{3}$ we are trying to find how many $\frac{8}{12}$ there are in $\frac{9}{12}$. This implies 9 twelfths divided by 8 twelfths. Now 8 twelfths into 9 twelfths we have I remainder 1 twelfth. The 1 twelfth is then written as a fraction of the divisor which is 8 twelfths. This is just 1 over 8. The answer is therefore $1\frac{1}{8}$ ,which can also be obtained dividing the numerators 9 and 8. In addition to providing access to visual representations, the common denominator method has other advantages that include: (1) reducing division of fractions to division of whole numbers (2) converting two denominators to a common denominator, a practice which pupils are familiar with, from addition and subtraction of fractions and (3) the relationship between division as an operation and fractions as quotients becomes clear to students (Toluk& Middleton, 2004).

## METHODS

### The Lowest Common Multiple

It is common practice to multiply or divide the numerator and the denominator of a fraction by the same number. This does not change the value of the fraction. In order to move from $\frac{3}{4} \div \frac{2}{3}$ to $\frac{9}{12} \div \frac{8}{12}$ both the numerator and the denominator of first fraction were multiplied by 3 and those of the second fraction were multiplied by 4. Looking closely at $\frac{3}{4}$ we see that it is a way of dividing whole numbers. This means that $\frac{3}{4}$ is the same as $3 \div 4$ hence $\frac{3\times3}{4\times3}$ is the same as $(3 \times 3) \div (4 \times 3)$. Multiplying the numerator and the denominator of a fraction by the same number is the same as multiplying the dividend and the divisor by that number. In $\frac{3}{4} \div \frac{2}{3}$, $\frac{3}{4}$ is the dividend and $\frac{2}{3}$ is the divisor. Multiplying each of the two fractions by the same number will not affect the value of the answer and this is a justified operation. We choose to multiply both fractions by the lowest common multiple of the denominators. Therefore $\frac{3}{4} \div \frac{2}{3}$ is the same as $\frac{3}{4} \times \frac{12}{1} \div \frac{2}{3} \times \frac{12}{1}$ and this reduces to $9 \div 8 = 1\frac{1}{8}$. By multiplying both the dividend and the divisor by the lowest common multiple of the denominators we reduce division of fractions to that of whole numbers, which pupils are comfortable operating with.

When this method is used this way, the name Lowest Common Multiple is the most suitable one. A shortcut of this method, which is being called Cross-Multiplication here is proposed below.

### Cross-Multiplication Approach

Division of fractions is written in general as $\frac{a}{b} \div \frac{c}{d}$. The lowest common multiple (LCM) of the denominators here is bd and from section 3, multiplying both the dividend and divisor by the LCM we have $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{\mathrm{bd}}{1} \div \frac{\mathrm{c}}{\mathrm{d}} \times \frac{\mathrm{bd}}{1}$. This reduces to $\mathrm{a} \times \mathrm{d} \div \mathrm{b} \times \mathrm{c}$. Looking closely at this we see that the dividend is the product of the numerator of the first fraction and the denominator of the second fraction, and the divisor is a product of the numerator of the second fraction and the denominator of the first fraction. This is where the name Cross-Multiplication comes from. Looking at our example, the next step after $\frac{3}{4} \div \frac{2}{3}$ is $3 \times 3 \div 2 \times 4 = 9 \div 8 = 1\frac{1}{8}$. Instead of multiplying both the dividend and the divisor by the LCM we just cross-multiply. This approach has a number of advantages over the conventional ‘invert and multiply’ approach. Cross-multiplication reduces the fractions to whole numbers which are easier to deal with. The operation remains division. Another key point, the cross-multiplication can be justified easily. The ‘invert and multiply’ on the other hand, you invert the divisor, change the operation and after all that, you are still dealing with fractions, whose operations are shunned by pupils. Worst of all, regardless of having been in use for centuries, no justification of the method has been provided to date.

## CONCLUSION

The ‘invert and multiply’ method, though widely used, is purely procedural and lacks conceptual depth. Its disadvantages include altering the operation while retaining fractions. The common denominator method provides a viable alternative with strong visual representation benefits. Additionally, the cross-multiplication method a conceptually justifiable and computationally efficient approach was introduced. This method reduces fraction division to whole number division, making it more accessible to learners. By adopting conceptually grounded methods, educators can enhance students' understanding of fraction division, fostering stronger mathematical reasoning skills.

## Recommendations

(1). Educators are encouraged to use the common denominator method in teaching, as it provides visual representations and fosters conceptual understanding.

(2). Teachers should learn and incorporate the cross-multiplication method along with its justification.

(3). Visual representation representations of fraction division should be a must teach as the enhance understanding.

(4). The 'invert and multiply' method should be discontinued, as it promotes memorization rather than comprehension.

## *Declaration by Authors*

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