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The Effect of Plate and Magnetic Field's Inclination on Fluid Temperature of an MHD Free Convective Poiseuille Flow

Dr. Richard Opiyo[1], Mbai J. Mutia[2]

[1,2]Department of Pure and Applied Mathematics, Maseno University, Kenya

Corresponding Author: Mbai J. Mutia

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ABSTRACT

This study investigated the effect of plate and magnetic field inclination on the fluid temperature of an MHD-free convective Poiseuille flow. The plates were inclined at an angle α to the fluid flow, while the magnetic field was inclined at an angle ξ to the y-axis. Besides, the fluid was assumed to be viscous, incompressible and could conduct electricity. A suitable mathematical model describing the problem was first formulated from the heat equation. The resultant model was a differential equation and was solved numerically using the shooting technique and the fourth-order Runge-Kutta method implemented in Mathematica Software. Results were graphed and interpreted. The temperature decreased as the magnetic angle of inclination, Eckert number, Prandtl number, and Hartmann number rose; a rising temperature accompanied an increasing Reynolds number. The results are applicable in industries where external magnet is used as a regulatory mechanism.

Keywords: Temperature, free convective, inclination, MHD, poiseuille flow, runge-kutta method.

1. INTRODUCTION

Magnetohydrodynamics (MHD) involves studying the macroscopic interaction of fluids that can con- duct electricity, including gases and liquids, with a magnetic field [4]. Here, fluid mechanics and electromagnetic equations describe the MHD flow. MHD draws from two main branches of physics: electrodynamics and hydrodynamics. The study of forces and the fluid flow they cause is known as electrodynamics. Whereas hydrodynamics is stated in conservation laws regarding mass, momentum, and energy. It is expressed in the form of Maxwell's equations. However, studies of fluid flow where parallel plates and magnetic fields are inclined simultaneously have yet to be explored. This is the foundation of the problem to be solved in our study. Including a magnetic field in a fluid flow affects the fluid temperature variation expressed using a magnetic parameter named Hartmann number, M. Hartmann number rises when the magnetic field intensity is increased [5]. Moreover, inclining a fluid flow channel introduces the influence of gravitational force parallel to the inclined plane pushing downward [11]. The buoyant force also determines the fluid flow caused by the free convection resulting from the temperature difference. The interaction between the Lorentz force and buoyant force governs the temperature fields.

The diverse applications of MHD flow in the design of many engineering devices, including MHD generators and accelerators, have made it a topic of interest for many researchers [2].

To date, MHD remains an affluent area of study. In their research, Reddy and Reddy [10] investigated the impact of heat generation and mass transfer on MHD-free convection flow through an inclined vertical plate. They discovered that the boundary layer's temperature and velocity rose whenever heat generation was raised. Ahmed [1] studied mass and Heat transmission in MHD Poiseuille flow in an evenly distributed magnetic field perpendicular to the plates. They found that temperature and concentration increased with the increase in Reynolds and Hartmann numbers. Chutia [3] investigated Poiseuille flow and transfer of heat in an inclined channel using a fluid that is viscous, incompressible, and conducts electricity. They found that the temperature field increased from the lower to the upper plate when the Eckert and Prandtl numbers increased.

2 Conceptualized Sketch of the Problem

Figure 1 below shows a conceptualized sketch of the problem in this study. It displays the direction of essential elements, including gravitational force (g), fluid flow, and magnetic induction (B_0). It also shows how the two-dimensional coordinate plane relates to other elements. In our study, we have two inclination angles, including the angle of inclination of the magnetic field (ξ) and the angle of inclination of the plate (α), as shown in the figure. We have two parallel plates shown in the figure separated by distance L where the lower plate's temperature is $T = T_0$ while the upper plate's temperature is $T = T_1$.

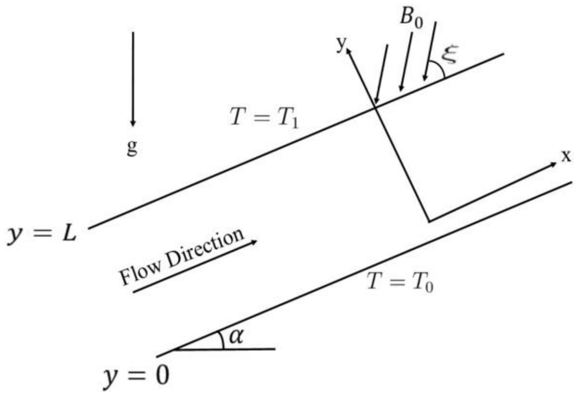

Figure 1: Geometry of the problem

3. NUMERICAL SOLUTION

The continuity equation, Maxwell's equations and energy equation are fundamental equations whose combination is utilized in explaining MHD flow.

The continuity equation is the partial differential equation, PDE, expressing mass conservation [7]. It comprises of only fluid density, ρ , and fluid velocity, V , and it is given by;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V = 0 \quad (1)$$

In this equation, t represents time, ∇ is the Gradient operator, ρ stands for the fluid's density and $\mathbf{V}$ represents the velocity component.

Electrically charged particles move parallel to local magnetic and electric fields due to Lorentz force. An electric field vector, denoted as **E**, is created perpendicular to both **V** and **B** when velocity and magnetic field interact. In this case, **V** stands for fluid velocity, and **B** represents the magnetic field. We have

$$\mathbf{E} = \mathbf{V} \times \mathbf{B} \quad (2)$$

For an isotropic fluid, the density of the induced current in the fluid designated by **J** is given by

$$\mathbf{J} = \sigma[\mathbf{V} \times \mathbf{B}] \quad (3)$$

σ denotes fluid's electrical conductivity.
Lorentz Force **F** co-occurs with induced current.

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} \quad (4)$$

Therefore, using (3) in (4) we have;

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} = \sigma(\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \quad (5)$$

Maxwell's equations express the generation and variation of electric and magnetic fields [6]. The equations are as follows;

$$\nabla \cdot \mathbf{B} = 0 \quad (6)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{D}}{\partial t} \quad (7)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (8)$$

Where **H** stands for the magnetic field vector, **D** stands for the electric displacement, and t stands for the time.

3.1. Energy Equation

The energy equation is essential in describing the energy conservation within our study's fluid system. It accounts for the kinetic, potential and internal energy [12]. The two-dimensional energy equation is given by;

$$0 = \frac{k}{\rho}\frac{\partial^2 T}{\partial x^2} + \frac{k}{\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mathbf{J}^2}{\partial \sigma} + \frac{\mu}{\rho}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\} \quad (9)$$

At this point, we considered the current intensity induced due to the fluid velocity in both x and y-direction.
From equation (3) given as $\mathbf{J} = \sigma(\mathbf{V} \times \mathbf{B})$. The current intensity **J** can be computed as follows;

$$\mathbf{J} = \sigma[(u, v, 0) \times \mathbf{B}] = \sigma \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & v & 0 \\ 0 & B_0 \sin\xi & 0 \end{vmatrix} = \sigma u B_0 \sin\xi \mathbf{k}$$

It then follows that

$$\mathbf{J}^2 = \sigma^2 u^2 B_0^2 \sin^2 \xi \quad (10)$$

Plugging equation (10) into (9) we obtain:

$$\frac{k}{\rho}\frac{\partial^2 T}{\partial x^2} + \frac{k}{\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma^2 u^2 B_0^2 \sin^2 \xi}{\partial \sigma} + \frac{\mu}{\rho}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right\} = 0 \quad (11)$$

with the associated boundary conditions

$$u = 0, T = T_0 \text{ at } y = 0 \tag{12}$$

$$u = 0, T = T_1 \text{ at } y = L \tag{13}$$

$$v = 0, T = T_0 \text{ at } y = 0 \tag{14}$$

$$v = 0, T = T_1 \text{ at } y = L \tag{15}$$

Thus, equation (11) is the formulated equation describing the fluid's temperature profile of the MHD flow past inclined plates in the presence of the inclined magnetic field.

3.2. Similarity Transformation Technique

The non-linear PDE, equation (11), was converted into ODE using the similarity transformation technique.
We define a 2-D stream function $\psi(x, y)$ such that it satisfies the continuity equation. Thus let; $u = \frac{\partial \psi}{\partial y}$, $v = \frac{\partial \psi}{\partial x}$ and η be the similarity variable, then $\theta(\eta)$ be the dimensionless temperature and U_∞ be the fluid's velocity away from the plate. Then, the non-dimension variables [9]

$$\eta = y\sqrt{\frac{U_\infty}{x\nu}}, \ \psi(x, y) = f(\eta)\sqrt{\nu U_\infty x}, \ \theta(\eta) = \frac{T - T_\infty}{T_1 - T_\infty} \tag{16}$$

(where ν is the kinematic viscosity and $f(\eta)$ is denotes the dimensionless stream function) were used to obtain a similarity solution. f and θ are functions of η.
From the stream function, we get the following transformation

$$\frac{\partial^2 T}{\partial y^2} = (T_1 - T_\infty)\frac{U_\infty}{x\nu}\theta'' = \frac{U_\infty}{x\nu}\theta''(T_1 - T_\infty) \tag{17}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{(T_1 - T_\infty)}{4x^2}(\eta^2\theta'' + 3\eta\theta') \tag{18}$$

From [8] let

$$u = U_\infty f', \quad \frac{\partial u}{\partial x} = -\frac{U_\infty}{2x}\eta f'', \quad \frac{\partial u}{\partial y} = U_\infty\sqrt{\frac{U_\infty}{x\nu}}f''$$

$$\frac{\partial u}{\partial x} = -\frac{1}{4x}\sqrt{\frac{\nu U_\infty}{4x^2}}[2\eta f' + \eta^2 f''], \quad \frac{\partial v}{\partial y} = \frac{U_\infty}{2x}[f' + \eta f'' \tag{19}$$

Using equations (17), (18) and (19), with $\nu = \frac{\mu}{\rho}$ in the energy equation (11) yields;

$$\frac{k(T_1 - T_\infty)}{4U_\infty^3\rho x}(\eta^2\theta'' + 3\eta\theta') + (T_1 - T_\infty)\frac{k}{U_\infty^2\rho\nu}\theta'' + \frac{\sigma B_0^2 x \sin^2 \xi f'f'}{U_\infty\rho} + \frac{\nu}{2U_\infty x}\eta^2 f''f'' +$$

$$\frac{\nu}{2U_\infty x}[f'f' + 2\eta f'f'' + \eta^2 f''f''] + \frac{\nu^2}{16U_\infty^2 x^2}[4\eta^2 f'f' + 4f'f'' + \eta^4 f''f''] -$$

$$\frac{\nu}{2U_\infty x}[2\eta f'f'' + \eta^2 f''f''] + f''f'' = 0 \tag{20}$$

Then let,

$$M = \frac{x\sigma B_0^2}{\rho U_\infty}, \ Re = \frac{U_\infty x}{\nu}, \ Pr = \frac{\nu\rho C_p}{k} \text{ and } Ec = \frac{U_\infty^2}{C_p(T_1 - T_\infty)} \tag{21}$$

substituting equation (21) in equation (20) we obtain;

$$\frac{3\,\eta\theta'}{4RePrEc} + \left[\frac{\eta^2}{4RePrEc} + \frac{1}{PrEc}\right]\theta'' + \left[M\sin^2\xi + \frac{1}{2Re} + \frac{\eta^2}{4Re^2}\right] f'f' + \left[\frac{\eta^3}{4Re^2}\right] f'f'' + \left[\frac{\eta^2}{2Re} + \frac{\eta^4}{16Re^2} + 1\right] f''f'' = 0 \tag{22}$$

Equation (22) is the energy equation to be solved using the boundary conditions below.

$$f = 0, \quad f' = 0, \quad \theta = 1 \ \text{at } \eta = 0$$
$$f' = 1, \theta = 0 \ \text{at } \eta = \infty \tag{23}$$

The obtained energy equation (22) is the formulated mathematical model that describes a two-dimensional steady magnetohydrodynamic free convective poiseuille heat transfer of a fluid past two infinite inclined parallel plates with the inclined magnetic field as in objective (1) of the study. Its solution gives the temperature profile of the study.

4. SIMULATION, RESULTS AND DISCUSSION

The shooting technique and the fourth-order Runge-Kutta method, implemented in Mathematica Soft- ware, were used to solve the nonlinear equations with their corresponding boundary conditions. In this case, the boundary value ODEs were converted using the shooting method technique utilizing Secant iteration into a group of first-order initial values for ODEs. The fourth-order Runge-Kutta technique, incorporated within Mathematica software, was then used to solve the resulting system. Temperature distributions were displayed in figures for various governing parameters to analyze the results of the numerical calculations. Physically realistic numerical values were assigned to the embedded parameters to help interpret the results.

Results were presented in graphs where an upward arrow in the graphs means that temperature increases with the increase of the corresponding parameter. In contrast, a downward arrow in the graphs means that temperature decreases with the rise of the corresponding parameter.

According to Figure (2), ξ affected Temperature distribution. Its increase decreased the temperature. There was a change in the Temperature when Ec was assigned different values; Temperature decreases with an increase in Ec as shown in (3). Hartmann number, M , affected the fluid temperature as displayed in Figure (4) in that its increase from 2 to 10 decreased the temperature. The temperature distribution reduced with an increase in Pr as displayed in Figure (5). According to Figure (6), Reynolds number increased the temperature when it was increased.

5. CONCLUSION

This study examined two-dimensional steady magnetohydrodynamic free convective Poiseuille heat transfer past an inclined magnetic field and two infinitely inclined parallel plates. The research aimed to determine the influence of combining inclined parallel plates and inclined magnetic fields on fluid temperature. Differential governing equations were formulated and solved using the the shooting tech- nique and the fourth-order Runge-Kutta method implemented in Mathematica Software. Results were

graphed and interpreted. The temperature profiles from the energy equations were graphed and tabulated to discuss the influence of magnetic field's inclination, and other fluid flow pertinent parameters on the fluid temperature.
According to the results, temperature is reduced with an increase in the magnetic angle of inclination, Eckert number, Prandtl number, and Hartmann number. The research results were essential as they would apply to the manufacturing industry, cooling of electrical equipment, nuclear reactor insulation, and crystal growth in liquids. In such industry, external magnet is used as a regulatory mechanism.

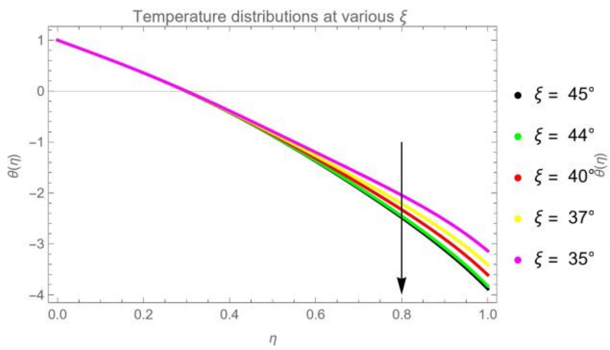

Figure 2: Temperature at various ξ

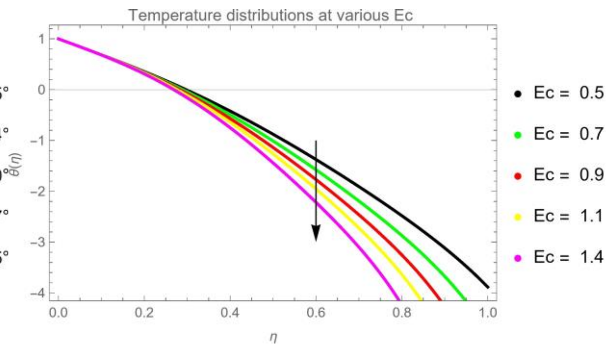

Figure 3: Temperature at various Ec

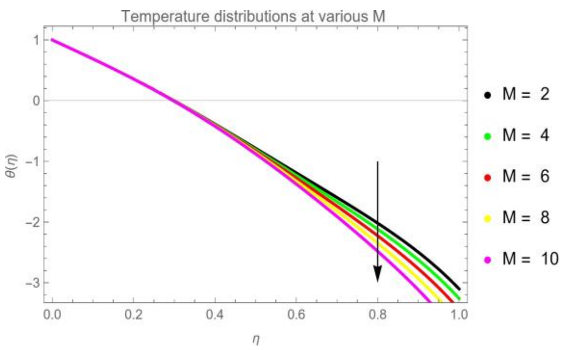

Figure 4: Temperature at various M

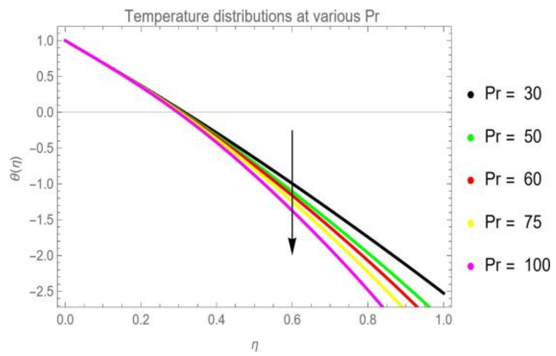

Figure 5: Temperature at various Pr

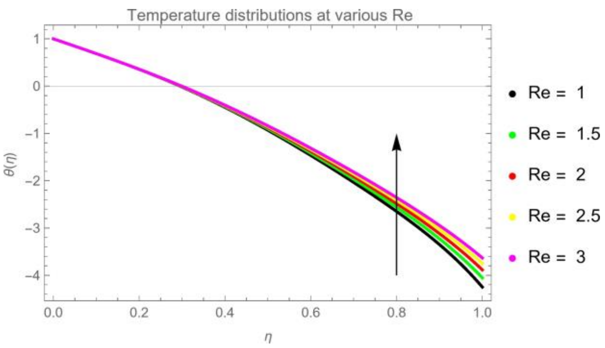

Figure 6: Temperature at various Re

Declaration by Authors

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